Hong Kong Mathematics Olympiad (2011 / 2012)
Heat Event (Group)
香港數學競賽 (2011 / 2012)
初賽項目(團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 已知 x、y 及 z 為三個連續正整數，且 $\frac{y}{x}+\frac{z}{x}+\frac{x}{y}+\frac{z}{y}+\frac{x}{z}+\frac{y}{z}$ 為整數，求 $x+y+z$ 的值。

   Given that x, y and z are three consecutive positive integers, and $\frac{y}{x}+\frac{z}{x}+\frac{x}{y}+\frac{z}{y}+\frac{x}{z}+\frac{y}{z}$ is an integer. Find the value of $x+y+z$.

2. 已知 x 是一個實數，且 $\sqrt{x-2012}+\sqrt{(5-x)^2}=x$，求 x 的值。

   Given that x is a real number and $\sqrt{x-2012}+\sqrt{(5-x)^2}=x$. Find the value of x.

3. 求 $\sqrt{2^2+2^{1008}+2^{2012}}$ 的值。(答案可以指數表示。)

   Evaluate $\sqrt{2^2+2^{1008}+2^{2012}}$. (Answer can be expressed in index form.)

4. 求 $\frac{1}{\sqrt{2012}+\sqrt{2011}}+\frac{1}{\sqrt{2011}+\sqrt{2010}}+\cdots\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{1}}$ 的值。(答案可以根式表示。)

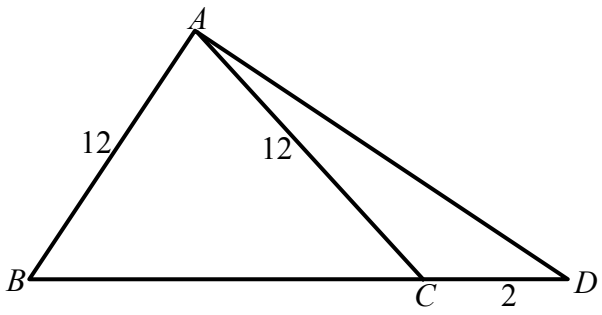
   Evaluate $\frac{1}{\sqrt{2012}+\sqrt{2011}}+\frac{1}{\sqrt{2011}+\sqrt{2010}}+\cdots\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{1}}$. (Answer can be expressed in surd form.)

5. 求 $x^2+y^2-10x-6y+2046$ 的最小值。

   Find the minimum value of $x^2+y^2-10x-6y+2046$.

6. 如圖三，ΔABC 為一等腰三角形。設 $AB = AC = 12$。若 D 是 BC 延伸線上的一點，使 $\angle DAB = 90°$ 及 $CD = 2$，求 BC 的長。

In Figure 3, ΔABC is an isosceles triangle. Suppose $AB = AC = 12$. If D is a point on the BC produced such that $\angle DAB = 90°$ and $CD = 2$, find the length of BC.



圖三
Figure 3

7. 已知 $a^x = b^y = c^z = 30^w$ 及 $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{w}$，當中 a、b、c 為正整數（$a \le b \le c$）及 x、y、z、w 為實數。求 $a+b+c$ 的值。

Given that $a^x = b^y = c^z = 30^w$ and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{w}$, where a, b, c are positive integers ($a \le b \le c$) and x, y, z, w are real numbers, find the value of $a+b+c$.

8. 已知方程 $x^2 + px + q = 0$ 的兩個根為整數，且 $q > 0$。若 $p + q = 60$，求 q 的值。

Given that the roots of the equation $x^2 + px + q = 0$ are integers and $q > 0$. If $p + q = 60$, find the value of q.

9. 求 $\sin^2 1° + \sin^2 2° + \sin^2 3° + \cdots + \sin^2 359° + \sin^2 360°$ 的值。

Evaluate $\sin^2 1° + \sin^2 2° + \sin^2 3° + \cdots + \sin^2 359° + \sin^2 360°$.

10. 在一集會中，原先安排每位賓客與其他賓客各握手一次，但小明只和他認識的人握手。如果集會中實際握手的總次數為 60 次，那麼小明在該集會中認識多少人？(註：當兩人相互握手，握手的總次數會是一次（而不是兩次）。)

    In a gathering, originally each guest will shake hands with every other guest, but Steven only shakes hands with people whom he knows. If the total number of handshakes in the gathering is 60 , how many people in the gathering does Steven know? (Note: When two persons shake hands with each other, the total number of handshakes will be one (not two).)

END